A Rigorous Frequency-Domain Approach to Large-Signal Noise in Nonlinear Microwave Circuits

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***Abstract*—The letter introduces a rigorous approach to large-signal noise in nonlinear microwave circuits. The analysis is based on the well-known representation of white noise as a set of uniformly spaced pseudosinusoidal signals with random phases. The spectral resolution required for a reliable noise simulation is evaluated by Monte Carlo analysis and is found to be quite high, so that the resulting nonlinear analysis involves spectra of many tens of thousands of lines. This kind of problem may be efficiently solved by the inexact-Newton harmonic-balance technique. With this method, the amplitude of the noise waveforms is arbitrary, so that large-signal noise effects may be directly evaluated.**

***Index Terms*—Harmonic balance, large-signal noise, nonlinear CAD.**

I. Introduction

SMALL-SIGNAL noise analysis in nonlinear microwave circuits is theoretically well established [1], [2], and is currently available in many state-of-the-art nonlinear simulators. The most commonly adopted (and most efficient) approach to this kind of analysis is to determine a large-signal unperturbed (noiseless) steady-state regime by the harmonic-balance (HB) technique, and then to evaluate the noise spectral densities of interest by first-order perturbation of such steady state. However, in many important applications, it may be impossible to treat noise power as being very small. A typical example is noise jamming in electronic countermeasures [3]: an accurate simulation of (say) a radar receiver in such conditions must rely upon the assumption that the useful signal and the jammer (noise) may have comparable strengths. Another important example is the computation of the noise power ratio (NPR) of a nonlinear power amplifier. NPR is often used as a representative measure of the intermodulation products generated by the amplifier in saturated multicarrier operation [4]; in such cases, the input noise spectral density must be large enough as to drive the amplifier well into the gain compression region.

The letter discusses a novel frequency-domain approach to the solution of this class of simulation problems. The analysis is based on the harmonic-balance (HB) technique and on the classic representation of band-limited white noise as a finite set of pseudosinusoidal signals uniformly spaced in frequency and with random phases [5]. Monte Carlo analysis is used to establish the minimum spectral resolution required to provide convergence of the results within a prescribed accuracy. It is found that the necessary spectral resolution is high, which may lead to spectra of several tens of thousands of lines, especially for highly nonlinear circuits such as saturated power amplifiers. In turn, this may result in HB problems of millions of nodal unknowns, which rules out conventional harmonic-balance methods. On the contrary, the problem can be efficiently solved by the recently proposed inexact-Newton harmonic-balance (INHB) technique [6]. This procedure is demonstrated by the computation of the power transfer characteristic and of the NPR of a saturated MESFET power amplifier.

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II. Large-Signal Noise Analysis

Let us consider a band-limited white noise having a constant power spectral density G_0 across a finite band B_ω centered around ω_0. G_0 is defined in such a way that $G_0 b_\omega$ represents the mean-square value of the noise voltage components whose spectrum falls inside an arbitrary band $b_\omega \subset B_\omega$. Now let the band B_ω be subdivided into a large number of equal intervals $\Delta\omega$, such that $\Delta\omega = B_\omega/(2H+1)$ where H is an integer. The center (angular) frequency of the hth interval $(-H \le h \le H)$ is then $\omega_0 + h\Delta\omega$. If $\Delta\omega$ is infinitesimal (i.e., for $H \to \infty$), the noise components whose spectrum falls inside the hth interval may be represented by a single pseudosinusoidal signal [5] of frequency $\omega_0 + h\Delta\omega$, having a random phase ϕ_h with uniform probability distribution in $[0 \quad 2\pi]$. The phases ϕ_h are statistically independent random variables [5]. For white noise, all such pseudosinusoids have a same amplitude $A = \sqrt{2G_0\Delta\omega}$. This leads to the following representation of the band-limited white noise voltage:

$$v_n(t) = \lim_{H\to\infty} \sqrt{\frac{2G_0B_\omega}{2H+1}} \cdot \sum_{h=-H}^{H} \cos\left[\left(\omega_0 + \frac{hB_\omega}{2H+1}\right)t + \phi_h\right]. \qquad (1)$$

In practice the computation of the limit is obviously impossible, so that (1) will be used with some large, but necessarily finite, value of H. The determination of H in such a way that the results of the analysis approach the limit to a prescribed accuracy represents an important aspect of the simulation problem.

Let us now assume that a signal expressed by (1) with a fixed finite value of H is processed by a nonlinear circuit, such as a mixer or a power amplifier, and that we are interested in the evaluation of all the intermodulation (IM) products of the pseudosinusoidal components of (1) up to the Mth order. A generic IM product has frequency

$$\sum_i p_i(\omega_0 + h_i\,\Delta\omega) = \omega_0 \sum_i p_i + \Delta\omega \sum_i p_i h_i \triangleq P\omega_0 + Q\Delta\omega \tag{2}$$

where p_i, h_i are integers with $\sum |p_i| \leq M$ and $|h_i| \leq H$. In turn, this implies $|P| \leq M$ and $|Q| \leq MH$. The signal (1) can obviously be interpreted as a carrier ω_0 modulated in amplitude and phase by a baseband pseudoperiodic signal of fundamental frequency $\Delta\omega$ and band $H\Delta\omega$. Similarly, due to (2), in order that the analysis may produce all of the IM products of interest, each waveform supported by the circuit must be described as a set of M carrier harmonics (plus d.c.), each modulated in amplitude and phase by a pseudoperiodic signal of fundamental frequency $\Delta\omega$ and band $MH\Delta\omega$. As an example, for a generic state variable this leads to the expression

$$x(t) = \mathrm{Re}\left[\sum_{m=0}^{M} \sum_{n=-MH}^{MH} X_{m,n} \exp\{j(m\,\omega_0 + n\,\Delta\omega)t\}\right]. \tag{3}$$

The signal (3) has $(2M+1)(2MH+1)$ degrees of freedom, because $X_{m,-n} \neq X_{m,n}{}^*, X_{0,-n} = X_{0,n}{}^*$ (* denotes the complex conjugate). Thus if the circuit state is identified by a total of n_S state variables, the overall number of scalar unknowns for the nonlinear simulation problem is given by $N_U = n_S(2M+1)(2MH+1)$.

With the representations (1) for the forcing terms (noise waveforms), and (3) for the problem unknowns (state variables), the noise analysis problem may be changed into an HB problem. In this way, the amplitude A, or equivalently the power spectral density G_0, may take on arbitrary values, so that large-signal noise problems may be directly tackled in the frequency domain. A rigorous approach to the problem should be based on Monte Carlo analysis in the following way. For a given value of H, a sequence of $2H+1$ random deviates representing the phases ϕ_h in (1) is extracted from a uniform probability distribution by means of a random number generator, and an HB analysis is carried out. The analysis is then repeated with different sequences of random phases, and the corresponding range of fluctuation of the results of interest (for the given H) is established. The whole procedure is then iterated for increasing values of H until the range of fluctuation becomes narrower than a prescribed threshold. In this process N_U may climb up to several tens of thousands or more, so that the use of conventional Newton-iteration-based HB algorithms is normally impossible due to the huge memory and CPU time required to solve the Newton equation. On the contrary, problems of this size are well within the reach of the INHB technique, since the storage and factorization of the Jacobian matrix are no longer necessary with this method [6]. In addition, with the INHB n_S equals the number of nonlinear device ports [6], which is always considerably smaller than the number of circuit nodes. This allows the available computer resources to be more effectively exploited than with iterative algorithms based on the nodal HB principle [7].

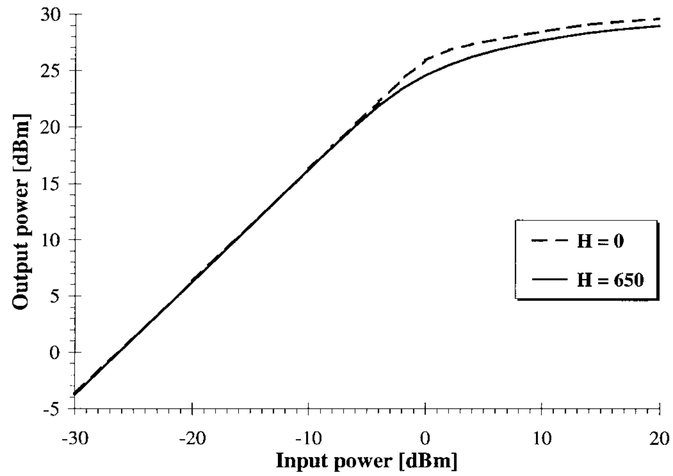

Fig. 1. Power saturation characteristic of a MESFET power amplifier driven by a sinusoidal signal and by white noise. The total input power in the 25-MHz band is the same in both cases.

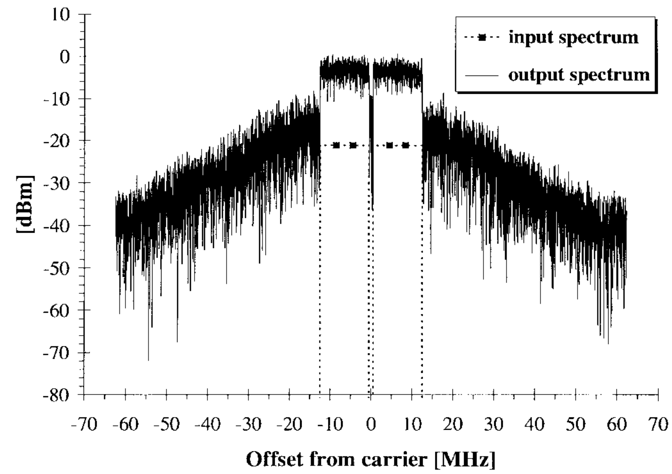

Fig. 2. Input and output noise spectral densities for the NPR calculation of a MESFET power amplifier ($H = 650$, $M = 5$). Input noise is white in the 25-MHz band except for a 1-MHz notch around center band.

III. Excitation of a Power Amplifier by Large-Signal Noise

Let us consider a single-stage class-A MESFET power amplifier operating in the 935–960-MHz band, with a saturated output power of about +30 dBm under single-tone sinusoidal excitation. Fig. 1 shows the I/O power characteristic of the amplifier driven by a sinusoidal signal and by white noise ($H = 650$), in such a way that the total input power in the 25-MHz band be the same in both cases. The analysis is carried out with $M = 5$. The figure shows that the saturated output power is only slightly degraded under the noisy excitation. Fig. 2 shows the input and output noise spectra used to compute the amplifier NPR at an input power level of +10 dBm, corresponding to an output power of

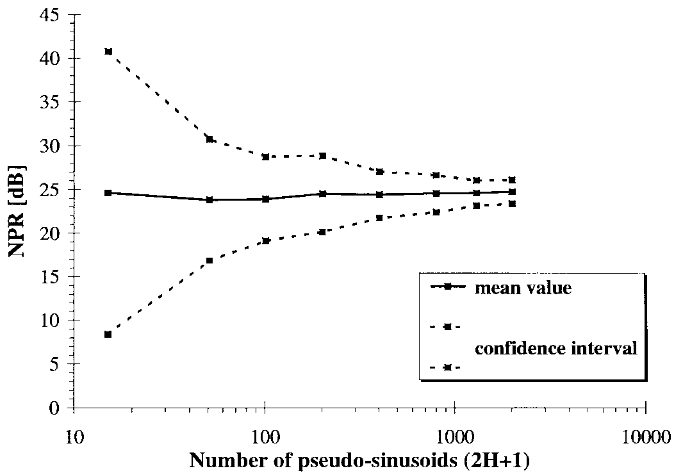

Fig. 3. Mean and confidence interval ($\pm 2.58\sigma$) of the amplifier NPR as a function of number of pseudosinusoids in the 25-MHz band, with constant input power $P_{\rm in} = +10$ dBm.

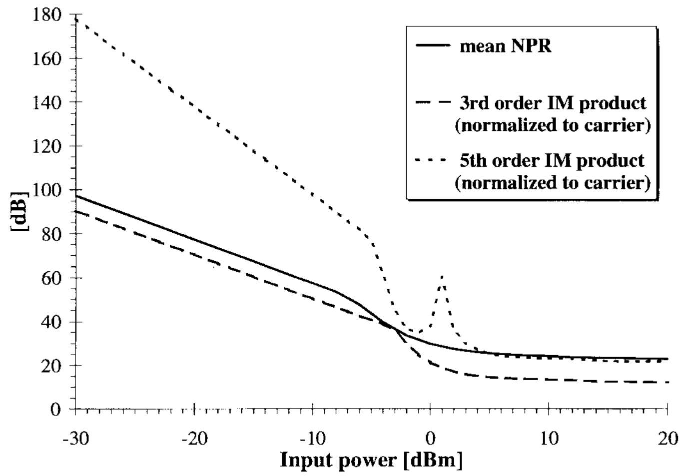

Fig. 4. Comparison between mean amplifier NPR and conventional third- and fifth-order near-carrier intermodulation products (generated by two tones of equal amplitudes spaced by 50 kHz).

about +28.4 dBm. The input white noise is suppressed in a 1-MHz notch around center band. On output, the NPR is defined as the ratio (in decibels) between the total noise power in the useful channel and the total power transferred to the notch due to intermodulation distortion in the amplifier. The computation reported in Fig. 2 is carried out with $H = 650$ and $M = 5$. Fig. 3 shows the results of a Monte Carlo analysis of the amplifier NPR. For a given H the NPR is a random variable depending on the specific sequence of carrier phases used in the simulation. Assuming that this random variable is normal, the probability that a single value extracted from its probability distribution (i.e., the NPR computed with an arbitrary sequence of carrier phases) falls within $\pm 2.58\sigma$ of its mean value m is about 99% (where σ^2 is the variance of the distribution). We shall thus define the confidence interval for the NPR as the range $[m - 2.58\sigma \; m + 2.58\sigma]$. In Fig. 3 the mean and the confidence interval evaluated from a statistical sample of 50 sequences are plotted against the number of pseudosinusoids (carriers), with a fixed total input power of +10 dBm (well inside the saturation region) in the 25-MHz band. The confidence interval is found to drop below 3 dB for $H \geq 650$. This shows that large values of H (for the present case $H \geq 650$, corresponding to a spectral resolution smaller than 19.2 kHz) may be required to provide an acceptably accurate representation of white noise. At the same time, Fig. 3 rigorously justifies the evaluation of the amplifier NPR as the mean of a number of samples (say, of the order of 50) generated by multicarrier excitations consisting of a relatively small number of equally spaced tones. Finally, Fig. 4 provides a comparison between the mean NPR and the ordinary third- and fifth-order IM products (normalized to the carrier power), evaluated under two-tone excitation as a function of input power. There is no apparent correlation among these three measures of IM distortion, although for the specific case under consideration the NPR turns out to be close to the fifth-order product in the saturation region.

For $H = 650$ and $M = 5$, a large-signal noise analysis requires a total of 35 755 spectral lines, so that the number of INHB unknowns is $N_U = 143\,022$. The analysis then requires about 341 MB of memory and about 5600 s of CPU time on a SUN Ultra 2 workstation (at +10-dBm total input power). Note that the circuit has 2 device ports and 15 nodes, so that the equivalent nodal problem would be one of 1 072 665 unknowns (over one million unknowns).

IV. Conclusion

The letter presents a general frequency-domain approach to large-signal noise in nonlinear microwave circuits. Noise is described as a set of pseudosinusoids according to classic models [5]. Harmonic-balance analysis with signal spectra including many tens of thousands of closely spaced lines that could generate numerical problems with conventional approaches [8] may be safely and efficiently tackled by the INHB technique [6]. With this method, noise can be treated rigorously and systematically just as any other input signal within the framework of general-purpose HB simulation. This represents a significant advance in nonlinear simulation capabilities for microwave engineering applications.

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